International Journal of Education and Science Research Review
ISSN 2348-6457
Volume-1, Issue-4
August- 2014
www.ijesrr.org
Email- editor@ijesrr.org

A Deterministic Inventory Model for a Deteriorating Item Is Explored In an Inflationary Environment for an Infinite Planning Horizon

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ABSTRACT:
In daily life, almost all the perishable products such vegetables, fruits dry fruits etc. lost their freshness day to day due to deterioration. Deterioration is (damage, spoilage, dryness etc.) is a very natural phenomena for everything in the world. In this paper, we have take two types of retailers whose demands are constant and stock dependent respectively. Here, shortages are taken but can't be fulfill deterioration is considered as constant. After calculating all the related costs we analyzed the total profit. Numerical examples have been presented to explain the theory.

Keywords: Inventory, stock dependent demand, constant deterioration, inflation and two types of retailers with constant.

INTRODUCTION:

In real life situations, the loss of inventory is due to demand as well as deterioration. Deterioration of many items during shortage period is a fact, such as chemical, IC chip, volatile liquids and so forth. It may be defined as the decay, damage, spoilage and evaporation of the stored items. Therefore, to maintain the inventory of deteriorating items is very important matter for any decision maker. There are several researchers who studied deteriorating inventory. Ghare and Schrader (1963) were the first proponent for developing a model with constant deterioration. Chung et. al. (2001) derived an inventory model for deteriorating items with the demand of linear trend and shortages during the finite planning horizon considering the time value of money. Mandal et al. (2006) discussed different types of inventory models for items with constant deterioration rate. Valliathal and Uthaya kumar (2009) considered a deterministic inventory model for perishable items under stock and time-dependent selling rate with shortages

Before 1975, the effect of inflation is disregarded in some inventory models, because it was considered that inflation does not have significant influence on the inventory policy.

However, the inventory system always needs to invest large capital to purchase inventory to obtain the high return of investment. Thus, this is very important aspect for any inventory system. Bazacott(1975) made the first attempt in this field that dealt with an EOQ model with inflation subject to different types of pricing policies. Ray et. al. (1997) developed a finite time-horizon deterministic economic order quantity (EOQ) inventory model with shortages under inflation, where the demand rate at any instant depends on the on-hand inventory (stock level) at that instant. Liao et al.(2000) presented a model with deteriorating items under inflation and permissible delay in payment. Chang(2004) considered an EOQ model with deteriorating items under inflation when the supplier credits are linked to order quantity. Maiti et al. (2006) developed an inventory model with stock-dependent demand and two storage facilities under inflation. The model is an order-quantity reorder-point problem where shortages are not allowed. S.RSingh ,Tarun Kumar,C.B. Gupta (2011) analyzed an Optimal Replenishment Policy for Ameliorating Item with Shortages under Inflation and Time Value of Money using Genetic Algorithm. Kapil Kumar Bansal, Anand (2013) Inventory Model for Deteriorating Items with the

Effect of inflation

However, most of the researches didn't take into account the effects of different retailers. There are so many practical situations where two classes of customers with different priority or waiting costs exist. There are many researches on inventory policies with two types of customers. Namias and Demmy (1981) treated two types of customers but non perishable products. Jang (2006) presented an integrated production and inventory allocation model in a two-echelon supply chain system. The higher echelon is a manufacturer, who produces a single commodity. The lower echelon consists of two types of major commodity distributors who might face stochastic or deterministic demands from multiple retailers. Singh, Chaman and Singh ,S.R. (2010) discussed two Echelon Supply Chain Model with Imperfect Production, for Weibull Distribution Deteriorating Items under Imprecise and Inflationary Environment. Firstly, Ishii (1993) introduced two types of customers to the prishable inventory models, which have different sensitivities to freshness of the products and obtained optimal ordering policies. Katagiri and Ishii (2002) discussed inventory control problems for a single perishable product with two types of customers, different selling prices, different holding costs, and shortage and outdating cost in a single-period horizon. It is a generalized model of Ishii (1993) in the sense that shortage and outdating costs are fuzzy numbers and holding costs are dependent upon the remaining lifetime of the product at the time of storage. Here we have taken a situation from the view point of a wholesaler. There are two retailers sell the same perishable product like fruit or seafood, which is ordered from the single wholesaler. Both of them place an order to the wholesaler to satisfy customer demands. One of the retailers is operated by the wholesaler whom we called self-operated chain store while the other is a franchised chain store of the wholesaler. Because of the uncertainty customer demand, while there is a discount of the purchase price, the wholesaler may purchase another more quantities of item besides the total ordering quantities of the two retailers. So there are excessive inventory of the wholesaler, then the self-operated chain store will be allocated more items than its initial order. The two retailers have different demand rates and different shortage cost when a shortage is allowed. Here the demand rate of franchised chain store and self operated chain store are constant and stock dependent respectively.

In this study, the problem of determining the optimal replenishment policy for deteriorating items with both constant and stock-dependent demands is considered. Here shortages are allowed without backlogging. Inflationary environment is taken. The total profit function is determined and optimal solution with respect to decision variable is carried out. Numerical example is also given.

Assumptions:

The following assumptions are taken:

(1) Replenishment is instantaneous and occurs at the beginning of each period.

(2) The lead time is zero.

(3)Shortage is allowed.

(4)Backlogging is not allowed.

(5)Inflation is taken.

Notations:

The following notations are used

T : replenishment time interval of the wholesaler;

t_1 : replenishment time interval of the inventory for the self-operated chain store;

t_2 : replenishment time interval of the inventory for the franchised chain store;

s : shortage cost rate per unit time,

s_1 is the shortage cost rate per unit time of self-operated chain store
s_2 the shortage cost rate per unit time of the franchised chain store;
A : ordering cost per order;
c : unit purchasing cost;
h : holding cost rate per unit time;
θ : constant deterioration rate, , where 0<θ <1;
I(t) : inventory level of the wholesaler at time t, where $I_1(t)$ is the inventory level of self-operated chain store and $I_2(t)$ is for the franchised chain store;
$D_1(t)$: demand rate of self-operated chain store is a linear function of on hand inventory level; where, $D_1(t)$ (t) =α + βI(t) , α > 0 and is a constant, 0< β <1;
$D_2(t)$: demand rate of franchised chain store, which is known and is a constant;
Q : ordering quantity of the wholesaler, where Q_1 is for self-operated chain store and Q_2 is for franchised chain store;
$\prod$: total profit of the wholesaler;
p_1 : selling price per unit to the customer from the self-operated chain store;
p_2 : selling price per unit to the franchised chain store from the wholesaler, where $p_1 > p_2$.
r: inflation rate

Model Formulation:
In this paper, we consider the whole inventory is divided into two parts after ordered. One is for the self-operated chain store and the other is for the franchised.

For self operated chain store: demand is given by

$$D_1(t) = \begin{cases} \alpha + \beta I_1(t); 0 \le t < t_1 \\ \alpha; t_1 \le t < T \end{cases}$$

Therefore the inventory level is given by differential equations

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -(\alpha + I_1(t)); 0 \le t < t_1 ------(1)$$

and

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = \alpha; t_1 \le t < T --------(2)$$

with boundary condition $I_1(t_1) = 0$.

For Franchised chain store demand is D_2. So, the differential equation is given by

$$\frac{dI_2(t)}{dt}+\theta I_2(t)=D_2;0\le t<t_2 ------(3)$$

$$\frac{dI_2(t)}{dt}=D_2;t_2\le t<T ------(4)$$

with boundary condition $I_2(t_2)=0$.

Now, the solutions of above differential equations, using boundary equations, are given by

$$I_1(t)=\frac{\alpha}{\beta+\theta}(e^{(\beta+\theta)(t_1-t)}-1);0\le t<t_1 -----(5)$$

and

$$I_1(t)=-\alpha(T-t);t_1\le t<T ------(6)$$

$$I_2(t)=\frac{D_2}{\theta}(e^{\theta(t_2-t)}-1);0\le t<t_2 -----(7)$$

and

$$I_2(t)=D_2(T-t);t_2\le t<T -----(8)$$

The ordering quantity is given by

$$Q_1=I_1(0)=\frac{\alpha}{\beta+\theta}(e^{(\beta+\theta)t_1}-1) -----(9)$$

And

$$Q_2=I_2(0)=\frac{D_2}{\theta}(e^{(\theta t_2}-1) -----(10)$$

The optimal ordering quantity Q^* of the wholesaler,

$$Q^*=I_1(0)+I_2(0)=\frac{\alpha}{\beta+\theta}(e^{(\beta+\theta)t_1}-1)+\frac{D_2}{\theta}(e^{(\theta t_2}-1) -------- (11)$$

Now we find all the related cost

Ordering Cost: the ordering cost during one cycle is fixed and constant, A. -------- (12)

Holding cost:

For self operated chain store is $C^h{}_1 = h\int_0^{t_1} I_1(t)e^{-rt}dt$

$$= h\int_0^{t_1} \frac{\alpha}{\beta+\theta}(e^{(\beta+\theta)(t_1-t)} - 1)e^{-rt}dt$$

$$C^h{}_1 = \frac{h\alpha}{2}t_1{}^2 \quad \text{--------} (13)$$

For franchised chain store is

$$C^h{}_2 = h\int_0^{t_2} I_2(t)e^{-rt}dt$$

$$= h\int_0^{t_2} \frac{D_2}{\theta}(e^{\theta(t_2-t)} - 1)e^{-rt}dt$$

$$C^h{}_2 = \frac{hD_2}{2}t_2{}^2 \quad \text{----------} (14)$$

Shortage cost:

For self operated chain store, shortage occurs during (t_1, T) .So, the total shortage cost is

$$C^s{}_1 = s_1\int_{t_1}^{T} -I_1(t)e^{-rt}dt$$

$$= -s_1\int_{t_1}^{T} -\alpha(T-t)e^{-rt}dt$$

$$C^s{}_1 = \alpha s_1(\frac{1}{2}(T-t_1)^2 + \frac{r}{2}t_1{}^2T) \quad \text{----------} (15)$$

For franchised chain store is

$$C^s{}_2 = s_2\int_{t_2}^{T} -I_2(t)e^{-rt}dt$$

$$= -s_2\int_{t_2}^{T} -D_2(T-t)e^{-rt}dt$$

$$C^s{}_2 = s_2D_2(\frac{1}{2}(T-t_2)^2 + \frac{r}{2}t_2{}^2T) \quad \text{----------} (16)$$

Purchasing cost

For self operated chain store

$$C^p{}_1 = cQ_1 = \frac{c\alpha}{\beta+\theta}(e^{(\beta+\theta)t_1} - 1)$$

$$= c\alpha(t_1 + \frac{(\beta+\theta)}{2}t_1{}^2) \quad \text{------------ (17)}$$

For franchised chain store is

$$C^p{}_2 = cQ_2 = \frac{cD_2}{\theta}(e^{(\theta t_2} - 1)$$

$$= cD_2(t_2 + \frac{\theta}{2}t_2{}^2) \quad \text{------------ (18)}$$

Sales revenue

For self operated chain store

$$R_1 = p_1\int_0^{t_1} D_1 e^{-rt}dt$$

$$= p_1\int_0^{t_1}(\alpha + \beta I_1(t))e^{-rt}dt$$

$$= p_1\int_0^{t_1}(\alpha + \beta\frac{\alpha}{\beta+\theta}(e^{(\beta+\theta)(t_1-t)} - 1))e^{-rt}dt$$

$$= \alpha p_1(t_1 + \frac{(\beta - r)}{2}t_1{}^2) \quad \text{------------ (19)}$$

For franchised chain store is

$$R_2 = p_2\int_0^{t_2} D_2 e^{-rt}dt$$

$$= p_2 D_2(t_2 - \frac{r}{2}t_2{}^2) \quad \text{-------------(20)}$$

Therefore, the **total profit** per unit time is

$$\prod(t_1, t_2, T) = \frac{1}{T}(R_1 + R_2 - A - C^h{}_1 - C^h{}_2 - C^s{}_1 - C^s{}_2 - C^p{}_1 - C^p{}_2)$$

$$= \frac{1}{T}(\alpha p_1(t_1 + \frac{(\beta - r)}{2}t_1{}^2) + p_2 D_2(t_2 - \frac{r}{2}t_2{}^2) - A - \frac{h\alpha}{2}t_1{}^2 - \frac{hD_2}{2}t_2{}^2 - \alpha s_1(\frac{1}{2}(T - t_1)^2 + \frac{r}{2}t_1{}^2 T)$$

$$- s_2 D_2(\frac{1}{2}(T - t_2)^2 + \frac{r}{2}t_2{}^2 T) - c\alpha(t_1 + \frac{(\beta+\theta)}{2}t_1{}^2) - cD_2(t_2 + \frac{\theta}{2}t_2{}^2))$$

$$\prod(t_1,t_2,T)=\frac{1}{T}\{(p_1-c)\alpha t_1+(\beta(p_1-c)-h-c\theta-s_1-p_1r)\frac{\alpha}{2}t_1^{\;2}+(p_2-c)D_2t_2$$
$$-(h+c\theta+s_2+p_2r)\frac{D_2}{2}t_2^{\;2}-A\}+s_1\alpha t_1+s_2D_2t_2-\frac{s_1\alpha r}{2}t_1^{\;2}-\frac{s_2D_2r}{2}t_2^{\;2}-\frac{\alpha s_1}{2}T-\frac{s_2D_2}{2}T$$

----------- (21)

Now, our objective is to maximize the profit function. Here, t_1, t_2 and T are decision variables.

From $\frac{\partial\prod(t_1,t_2,T)}{\partial t_1}=0$, $\frac{\partial\prod(t_1,t_2,T)}{\partial t_2}=0$ and $\frac{\partial\prod(t_1,t_2,T)}{\partial T}=0$

We get

$$\frac{1}{T}\{(p_1-c)\alpha+(\beta(p_1-c)-h-c\theta-s_1-p_1r)\alpha t_1\}+s_1\alpha-s_1\alpha rt_1=0$$ -------- (22)

$$\frac{1}{T}\{(p_2-c)D_2-(h+c\theta+s_2+p_2r)D_2t_2\}+s_2D_2-s_2D_2rt_2=0$$ ---------(23)

and

$$-\frac{1}{T^2}\{(p_1-c)\alpha t_1+(\beta(p_1-c)-h-c\theta-s_1-p_1r)\frac{\alpha}{2}t_1^{\;2}+(p_2-c)D_2t_2$$
$$-(h+c\theta+s_2+p_2r)\frac{D_2}{2}t_2^{\;2}-A\}-\frac{\alpha s_1}{2}-\frac{s_2D_2}{2}=0$$

---------- (24)

To maximize total average profit per unit time $\prod$ (t_1, t_2, ,T) the optimal values of t_1 and t_2 and T can be obtained by solving the above equations (22), (23) and (24) simultaneously. Provided, they satisfy the following conditions of optimality (i.e. H will be negative and all the entries of H are positive.)

$$H=\begin{bmatrix}\frac{\partial^2\prod(t_1,t_2,T)}{\partial t_1^{\;2}} & \frac{\partial^2\prod(t_1,t_2,T)}{\partial t_1\partial t_2} & \frac{\partial^2\prod(t_1,t_2,T)}{\partial t_1\partial T}\\ \frac{\partial^2\prod(t_1,t_2,T)}{\partial t_2\partial t_1} & \frac{\partial^2\prod(t_1,t_2,T)}{\partial t_2^{\;2}} & \frac{\partial^2\prod(t_1,t_2,T)}{\partial t_2\partial T}\\ \frac{\partial^2\prod(t_1,t_2,T)}{\partial T\partial t_1} & \frac{\partial^2\prod(t_1,t_2,T)}{\partial T\partial t_2} & \frac{\partial^2\prod(t_1,t_2,T)}{\partial T^2}\end{bmatrix}$$

Here, to find the values of t_1, t_2 and T and for the condition of optimality, we use the software

CONCLUSION:

We develop a model for an optimal ordering policy for deteriorating items with two retailers, where shortage is allowed but backlogging is not. The two retailers have different shortage cost and demand rate. One is constant demand and the other is stock-dependent demand. We discuss about the concavity conditions of the wholesaler's profit function. Simple algorithms have been provided to find the optimal replenishment timing for the proposed model. In this paper deterioration is taken as constant. Inflationary environment is also taken to be realistic. This is very important factor for any business in the world.

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